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Book Review: *The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations*

The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations. L. E. Reichl, Springer-Verlag, Berlin, 1992.

There are many different manifestations of the probabilistic behavior of physical objects. These may be consequences either of inherent quantum properties or they may be consequences of some form of the law of large numbers in many-body systems. Sometimes one needs an external random force to produce chaotic behavior of a system, and sometimes this behavior is a basic property of nonlinear dynamical systems. This possibility was discovered about 25 years ago in the context of classical mechanics, hitherto thought to be quite simple from the mathematical point of view. Such "simplicity" is, of course only a seeming simplicity. Newton's laws are by no means self-evident, and even can contradict a commonsense point of view. Indeed, why does a body continue to move with a constant velocity when no external force acts on the body? In other words, why is the force proportional to the acceleration rather than to the velocity? We are already used to this and other paradoxical features of mechanics. Only in recent times has the new and somewhat surprising phenomenon of deterministic chaos forced itself to the attention of scientists in a number of disciplines. Such chaos appears not only in some nonlinear classical systems, but also in a number of quantum systems. A clear description of the onset of chaos in classical nondissipative systems and its quantum manifestations is the subject of the book being reviewed.

Reichl's book consists of three parts, describing the dynamics of conservative systems, both classical (Chapters 2–4) and quantum mechanical (Chapters 5–9), in addition to a number of stochastic properties of classical systems (Chapter 10). Chapter 2 contains a general description of chaos based on the concept of nonlinear resonances, and includes the Noether and KAM theorems. Three illustrative examples are given; the three-body Toda lattice, the two-resonance Walker–Ford Hamiltonian, and the conservative Duffing oscillator. Chapter 3 describes area-preserving twist maps, including the linearized mapping (tangent map), the whisker map and its local version (standard map), as well as the universal and quadratic

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maps. Frequency doubling and diffusion in two-dimensional maps are considered in some detail. Global properties of the maps considered in the previous section are analyzed in Chapter 4 starting from a Hamiltonian formulation. Interactions between different resonances are analyzed by the Chirikov overlap criterion and by renormalization group mapping. The chapter also contains a description of Arnold diffusion in systems with more than two degrees of freedom.

Chapter 5 is devoted to the concept of integrability in quantum systems based on Moyal brackets, quantum Lax pairs, and Peres' time averaging. Chapter 6 discusses Hamiltonian random matrix theory, allowing one to formulate the changes in the spectral properties of nonintegrable quantum systems. The Δ_3-statistics considered at the end of the chapter gives an additional tool for detecting differences between integrable and nonintegrable quantum systems. Chapter 7 is devoted to a consideration of energy spectra obtained both experimentally and numerically with a Poisson-like or Wigner-like spectrum. Billiard balls of different shapes and two anharmonic oscillator systems are used to illustrate the appearance of quantum chaos. Chapter 8 discusses the uses of semiclassical path integrals and the Gutzwiller trace formula to obtain the spectral properties of quantum systems, with billiard trajectories and the anisotropic Kepler system as examples. Periodically driven quantum systems are described in Chapter 9, where the appearance of quantum nonlinear resonances and their interactions are discussed. Examples here include the quantum kicked oscillator and the microwave-driven hydrogen atom. The final chapter covers the subject of how chaos is manifested in stochastic systems described by a Fokker–Planck equation, which has been discussed by the author and her collaborators in the literature. A few appendices contain useful information exemplified by the transition from coordinate–momentum to action–angle variables in a few illustrative examples, Moyal brackets, $SU(3)$ and space-time symmetries, and generating functions for Gaussian and circular orthogonal ensembles.

There are a number of earlier books on this subject (e.g., F. Haake, *The Quantum Signature of Chaos*, and M. Gutzwiller, *Chaos in Classical and Quantum Mechanics*, both published by Springer-Verlag in 1990). The present volume makes extensive reference to the scientific literature up to 1991, which is quite useful to scientists working in chaos and related fields. Reichl's book is a good example of a systematic and nonchaotic description of chaos.

Moshe Gitterman
Department of Physics
Bar-Ilan University
Ramat-Gan, Israel